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CAMERON STATION. ALEXANDRIA. VIRGINIA

63-3-2

ANALYSIS OF THE DYNAMICS OF ELASTIC SYSTEMS

Contract No. AF 49(638)-962

FINAL REPORT

1 November 1960 - 31 October 1962

1. **Summary of completed work.** Research activities during this period have been largely concentrated in two areas: the application of asymptotic methods to problems of elastic vibration and wave motion and the analysis of stochastic effects in the behavior of elastic structures.

Two problems in elastic vibrations have been solved. One is concerned with the vibrations of a pendulum in which the elastic properties of the support rod are taken into account. The method of solution is based on an asymptotic expansion in terms of the small stiffness of the rod. Similarly, the transverse vibrations of a circular plate under tension have been studied. A related problem which has been analyzed is the elastic stability of bars in which the cross-sectional dimensions are not small when compared with the length. The first problem in a series of investigations of elastic wave motion has been completed with a study of the surface waves produced on a slightly curved elastic half-space.

In the area of random effects, an analysis of the bending of columns by axial loads of random eccentricity has been completed. On the basis of assumptions concerning the statistical

distributions of the eccentricities of loading, information is obtained about the probable location of maximum transverse displacement and about the statistical relation between the applied load and the resulting midspan displacement. The transverse vibrations of a taut string with randomly varying mass distribution has been investigated. A somewhat similar problem concerning the longitudinal vibrations of an elastic bar whose cross-sectional area varies in a random manner has been solved.

2. **Work in progress.** Work is continuing on the diffraction and reflection of waves in an elastic medium. Particular emphasis is being placed on the development and use of perturbation and asymptotic methods. Further work on the effects of random variations in geometry and boundary conditions on the free vibrations of one dimensional elastic structures is in progress. Integral equations techniques as well as perturbation and asymptotic methods will be employed to estimate important statistical parameters associated with the natural frequencies of vibrations.

3. **Personnel.** The principal investigators under this contract have been George H. Handelman (Professor and Chairman of the Department of Mathematics) and William E. Boyce (Associate Professor of Mathematics).

Graduate assistants who have worked on these investigations are Charles J. Martin, Bruce E. Goodwin, and Robert D. Sidman. Dr. Martin received his Ph.D. degree in Mathematics in June 1961. His

work during his last year at Rensselaer was supported by this contract. His doctoral research was supervised by Professor Handelman and a portion of his thesis will be published as indicated below.

Mr. Goodwin has been supported by the contract for two years, and is presently completing work on his doctoral dissertation under the supervision of Professor Boyce.

Mr. Sidman presently holds a National Defense Education Act fellowship and is therefore not directly supported by the contract. His research work is, however, germane to the subject of the contract and is supervised by Professor Handelman.

4. **Publications**. The following publications have received support under the contract:

1. William E. Boyce, "The bending of columns under axial loads of random eccentricity", AFOSR TN - 703, Proceedings of the Seventh Midwestern Mechanics Conference, East Lansing, Michigan, 1961, pp. 143-152.

2. William E. Boyce, "Random Vibration of Elastic Strings and Bars", AFOSR 1908, Proceedings of the Fourth U.S. National Congress of Applied Mechanics, Berkeley, Cal., 1962 (in press)

3. George H. Handelman and F. F. Ling, "Elastic Stability of Bars of Arbitrary Length", AFOSR 2264, Proceedings of the Fourth U.S. National Congress of Applied Mechanics, Berkeley, Cal., 1962, (in press)

4. George H. Handelman and J. B. Keller, "Small Vibrations of a Slightly Stiff Pendulum", AFOSR 2385, _Proceedings of the Fourth U. S. National Congress of Applied Mechanics_, Berkeley, Cal., 1962 (in press)

5. Charles J. Martin, "Transverse Vibrations of a Circular Plate under Tension", _Proceedings of the Fourth U.S. National Congress of Applied Mechanics_, Berkeley, Cal., 1962(in press)

6. George H. Handelman, "Surface Waves on a Slightly Curved Elastic Half-space", (to appear in Progress in Applied Mechanics)